Enumeration and approximation of the efficient solutions in multi-objective combinatorial optimization problems

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Abstract: This thesis deals with the resolution of multi-objective combinatorial optimization problems. Classically, a first step in the resolution of these problems consists in determining the set of efficient solutions. Nevertheless, the number of efficient solutions can be very huge. Approximating the set of efficient solutions for these problems constitutes, then, a major challenge. Existing methods are usually based on approximate methods, such as heuristic or meta-heuristic, that give no guarantee on the quality of the computed solutions. Alternatively, approximation algorithms (with provable guarantee) have been also designed. However, practical implementations of approximation algorithms are cruelly lacking and most of the approximation algorithms proposed in the literature are not efficient in practice.

This thesis aims at designing approaches that conciliate on the one hand the qualities of the approximate approaches (efficiency in practice) and on the other hand those of the approximation approaches (guarantee *a priori* in terms of: running time, quality and number of computed solutions). For this purpose, we propose, in a general context, where the preference relation used to compare solutions is not necessarily transitive, a Generalized Dynamic Programming framework. This framework relies on an extension of the concept of dominance relations. It provides us, in particular, with exact and approximation methods that have been proved to be particularly efficient in practice to solve the 0–1 multi-objective knapsack problem. Moreover, we studied in the bi-objective case the properties to be satisfied by a set of solutions returned by an approximation method. Finally, a last part of our work deals with the contributions of a multi-criteria modelling for solving, in a real-world context, the data association problem, stated as a multi-objective multi-dimensional assignment problem.

Keywords: multi-objective combinatorial optimization, approximation with provable guarantee, exact resolution, dynamic programming, dominance relations, 0–1 multi-objective knapsack problem, multi-objective assignment problem.