Thesis Topic : *Extension of pre-solutions in combinatorial problems.*

Location : *LAMSADE, université Paris-Dauphine, Paris, France*

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Key words : Complexity and parametrised complexity, approximation, graph classes

Financing : Priority topic which could be funded by a Doctoral Contract of the Doctoral School of the University Paris Dauphine. Fully funded for 3 years. There is no obligation to teach, but the possibility to teach (with additional payment). Starting date is expected in September-October 2018.

Description of the topic :

Most of the problems in combinatorial optimisation are quite difficult from the computation point of view. One of the principal objectives of modern theoretical computer science consists of overcoming this algorithmic barrier by proposing either approximate solution or by providing additional information on the assumptions. The objective of this thesis is to study the impact on the complexity (or parameterized complexity) of a problem under the additional condition of extending a particular partial solution. The extension of pre-solutions is already present in diverse techniques of computer science : a search tree algorithm which solves an optimisation problem is often based on the extension of solutions in order to prune the branches of the search tree as soon as possible. Also, for example, for the problem of Vertex Cover (where one tries to cover the edges of a graph with a small number of vertices), it is desirable to be able to quickly (i.e., in polynomial time) decide whether a subset of vertices (which we call a pre-solution) can form a part of a minimum solution. This type of consideration remains interesting even in the extreme case of deciding whether a particular vertex belongs to an optimal cover. Moreover, this approach is already used when one wishes to enumerate the set of minimum covers or more generally the set of minimum solutions of an optimisation problem. [6, 7]. Finally, the extension of pre-solutions play a leading role in the techniques based on dynamic programming. There exists in the literature several articles considering the extension of pre-solutions, but in a sporadic manner [1, 3, 4, 5, 8, 8, 9]. Very recently, in [2] and [10] two theoretical frameworks had been proposed for the study of extensions of pre-solutions. In these frameworks, it is assumed that the problems of optimisation are well-structured, namely, there exists a partial order on the set of realisable solutions. For example, for *hereditary* problems (resp. *anti-hereditary*), the underlying order is inclusion (resp., exclusion). In the case of inclusion, the objective is to extend a part of the solution in a manner satisfying a given criteria of robustness compatible with the partial order. The most natural criteria to satisfy is the *Pareto Dominance* [2], which is minimality in our context, that is the removal of elements of a solution no longer guarantees its realisability. On the other hand, the most difficult criteria to verify is *optimality* or *quasi-optimality* [10, 8].

In this thesis, we wish to continue studying these two frameworks for other well known combinatorial problems and extending them to the other types of partial orders on the set of realisable solutions such as lattices.

Conditions :

The candidate must meet the following conditions
— A master degree either in mathematics or computer science, ideally with specialisation either in algorithms or combinatorial optimisation
— A good knowledge of complexity theory, approximation algorithms and graph theory
— Good knowledge of English (written and spoken)

Applications :

The applications with the required documents must be sent to : `jerome.monnot@dauphine.fr` et `Ararat Harutyunyan` before the end of March 2018.

The following documents must be attached to the application :

— A short letter of motivation
— A curriculum vitae ;
— Transcripts (including that of the Master degree studies). Bachelors and Master's diplomas
— Two letters of recommendation if possible.

A second selection will be carried out by the PhD program and the laboratory based on an interview on May 7th, 2018.

Références

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